AP Calculus BC

Intro to Sequences – Day 2

1) $\lim_{x\to\infty}\left(1-\frac{1}{x}\right)^{11x}$

$\lim_{x\to\infty}\left(1-\frac{11}{11x}\right)^{11x} = \boxed{e^{-11}}$

2) $\lim_{x\to 1}\frac{\int_1^x \cos(5t)\,dt}{x^2-1} = \frac{0}{0}$

$\lim_{x\to 1}\frac{\cos(5t)}{2x} = \frac{\cos 5}{2}$

3) $\lim_{n\to\infty}\left(1+\frac{2}{n}\right)^n = e^2$

4) $\lim_{n\to\infty}\sqrt{\left(1-\frac{1}{10n}\right)^n}$

$\sqrt{\lim_{n\to\infty}\left(1+\frac{-1/10}{n}\right)^n} = \sqrt{e^{-1/10}}$

$= e^{-1/20}$

5) $\lim_{n\to\infty}\left(1+\frac{9}{n}\right)^{8n}$

$\lim_{n\to\infty}\left(1+\frac{72}{8n}\right)^{8n} = \boxed{e^{72}}$

6) $\lim_{n\to\infty}\frac{n}{e^{n+11n}} = 0$

7) $\lim_{n\to\infty}\left(\frac{10}{n}\right)^{10/n} = 0^0$

$y = \left(\frac{10}{n}\right)^{10/n}$

$\ln y = \frac{10}{n}\ln\left(\frac{10}{n}\right)$

$= \lim_{n\to\infty}\left[\frac{10}{n}\ln\left(\frac{10}{n}\right)\right] = 0\cdot(-\infty)$

$= \lim_{n\to\infty}\frac{\ln\left(\frac{10}{n}\right)}{n/10} = \frac{-\infty}{\infty}$

L'Hopital's Rule

$= \lim_{n\to\infty}\frac{-1/n}{1/10} = 0$

$e^0 = 1$

8) $\ln(n^3+5) - \ln(7n^3+11n)$

$\ln\frac{(n^3+5)}{(7n^3+11n)}$

$\lim_{n\to\infty}\left[\ln\frac{n^3+5}{7n^3+11n}\right] = \boxed{\ln 1/7}$

9) $\lim_{n\to\infty}\left(n\sin\frac{17}{n}\right) = \infty\cdot 0$

$\lim_{n\to\infty}\frac{\sin\left(\frac{17}{n}\right)}{1/n} = \frac{0}{0}$

$\lim_{n\to\infty}\frac{\cos\left(\frac{17}{n}\right)\cdot -17/n^2}{-1/n^2} = \boxed{17}$

10) $\lim_{n\to\infty} \frac{(-1)^n n}{2n+3}$

DNE by oscillation

11) $a_n = \cos\left(\frac{n\pi}{12}\right)$

$\lim_{n\to\infty} \cos\left(\frac{n\pi}{12}\right)$ DNE

12) $\lim_{n\to\infty} \frac{(-1)^{n+1}}{4n+1} = 0$

Not Monotonic

Monotonic → always increasing or always decreasing

13) $a_n = \frac{3^n}{3^n+5^n}$

$\lim_{n\to\infty} a_n = 0$

14) $\lim_{n\to\infty} \left(2+\cos\frac{5}{n}\right) =$

$2 + \cos 0 = 3$

15) $\lim_{n\to\infty} \frac{\ln n}{n^{1.6}} = 0$

16) $\lim_{n\to\infty} \frac{8^n+5^n}{8^n+n^{110}} = 1$

17) $a_n = \frac{9^n}{n^9 7^n}$

$\lim_{n\to\infty} a_n$ DNE